Stuff you should already know

by ninth grade

Everything you learn in math you will use again and again.

Immediate, Automatic, Effortless

seven

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$6+1$

$5+2$

$4+3$

Immediate, Automatic, Effortless

seven

$$6 + 1$$

$$5 + 2$$

$$4 + 3$$

Almost everything the brain does is done subconsciously.

You just need to practice until you no longer have to think about it.

Counting

All of mathematics is based on counting.

$$
\begin{aligned}
2 &= 1+1 \\
3 &= 2+1 &= 1+1+1 \\
4 &= 3+1 &= 1+1+1+1 \\
5 &= 4+1 &= 1+1+1+1+1 \\
6 &= 5+1 &= 1+1+1+1+1+1 \\
7 &= 6+1 &= 1+1+1+1+1+1+1 \\
8 &= 7+1 &= 1+1+1+1+1+1+1+1 \\
9 &= 8+1 &= 1+1+1+1+1+1+1+1+1 \\
10 &= 9+1 &= 1+1+1+1+1+1+1+1+1+1
\end{aligned}
$$

therefore

$$
\begin{aligned}
2+2 &= (1+1)+(1+1) \\
&= 1+1+1+1 \\
&= 4
\end{aligned}
$$

The order by which numbers are counted **defines** the numbers.

Addition and Subtraction

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

If you can count, you can add and subtract.

Multiplication and Division

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

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Negative Numbers

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The product of two negatives is positive.

$$(-a)(-b) = ab$$

Negative Numbers

When subtracting a large number from a small number, you can pull the sign outside and reverse the subtraction.

$$a - b = -(b - a)$$

Reciprocals

Dividing is the same as multiplying by a reciprocal.

$$x \div \left(\frac{a}{b}\right) = x\left(\frac{b}{a}\right)$$

$$a/b = a\left(\frac{1}{b}\right)$$

Fractions

Addition (with common denominators):

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

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Multiplication:

$$\frac{a}{c} \times \frac{b}{d} = \frac{ab}{cd}$$

Addition (cross product):

$$\frac{a}{c} + \frac{b}{d} = \frac{ad+bc}{cd}$$

Decimals

$$12.34 \quad = \quad (1 \times 10) + (2 \times 1) + (3 \times \frac{1}{10}) + (4 \times \frac{1}{100})$$

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$$
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&= 1234/100
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$$12.34 \times 10 = 123.4$$

Dividing by 10 moves the decimal to the left 1 place.

$$12.34/10 = 1.234$$

Powers of 10

$$
\begin{array}{rcl}
& \vdots & \\
10^{-3} & = & 0.001 \\
10^{-2} & = & 0.01 \\
10^{-1} & = & 0.1 \\
10^{0} & = & 1 \\
10^{1} & = & 10 \\
10^{2} & = & 100 \\
10^{3} & = & 1000 \\
& \vdots &
\end{array}
$$

Powers of 10

$$\begin{aligned} &\vdots \\ 10^{-3} &= 0.001 \\ 10^{-2} &= 0.01 \\ 10^{-1} &= 0.1 \\ 10^{0} &= 1 \\ 10^{1} &= 10 \\ 10^{2} &= 100 \\ 10^{3} &= 1000 \\ &\vdots \end{aligned}$$

Scientific Notation:

$$a \times 10^n \quad \text{where} \quad 1 \leq a < 10$$

Squares and Square Roots

$$\begin{aligned}
1^2 &= 1 \\
2^2 &= 4 \\
3^2 &= 9 \\
4^2 &= 16 \\
5^2 &= 25 \\
6^2 &= 36 \\
7^2 &= 49 \\
8^2 &= 64 \\
9^2 &= 81 \\
10^2 &= 100
\end{aligned}
\qquad\qquad
\begin{aligned}
\sqrt{1} &= 1 \\
\sqrt{4} &= 2 \\
\sqrt{9} &= 3 \\
\sqrt{16} &= 4 \\
\sqrt{25} &= 5 \\
\sqrt{36} &= 6 \\
\sqrt{49} &= 7 \\
\sqrt{64} &= 8 \\
\sqrt{81} &= 9 \\
\sqrt{100} &= 10
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Exponents

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$$
\begin{aligned}
x^1 &= x && \\
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$$\begin{aligned} (ab)^n &= a^n b^n \\ x^n x^m &= x^{(n+m)} \\ (x^n)^m &= x^{nm} \end{aligned}$$

Prime Numbers

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$$\{2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots\}$$

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Prime Factorization

Any whole number > 1 can be represented as the product of prime numbers. Simply work through the list of prime numbers, seeing which ones divide evenly.

$$\begin{aligned} 60/2 &= 30 \\ 30/2 &= 15 \\ 15/3 &= 5 \end{aligned}$$

$$60 = 2^2 \times 3 \times 5$$

Sets

Standard sets of numbers include

$$\begin{aligned}
\emptyset &= \text{null set} \\
\Re &= \text{real numbers} \\
\{1, 2, 3, \ldots\} &= \text{natural numbers} \\
\{0, 1, 2, \ldots\} &= \text{whole numbers} \\
\{\ldots, -3, -2, -1\} &= \text{negative numbers} \\
\{\ldots, -2, -1, 0, 1, 2, \ldots\} &= \text{integers}
\end{aligned}$$

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Irrational numbers are everything left in $\Re$ after removing rational numbers. They include things such as π and $\sqrt{2}$.

Set Operations

$x \in A$	x is an element of set A
$A \subset B$	set A is a subset of set B
$A \cup B$	the union of sets A and B
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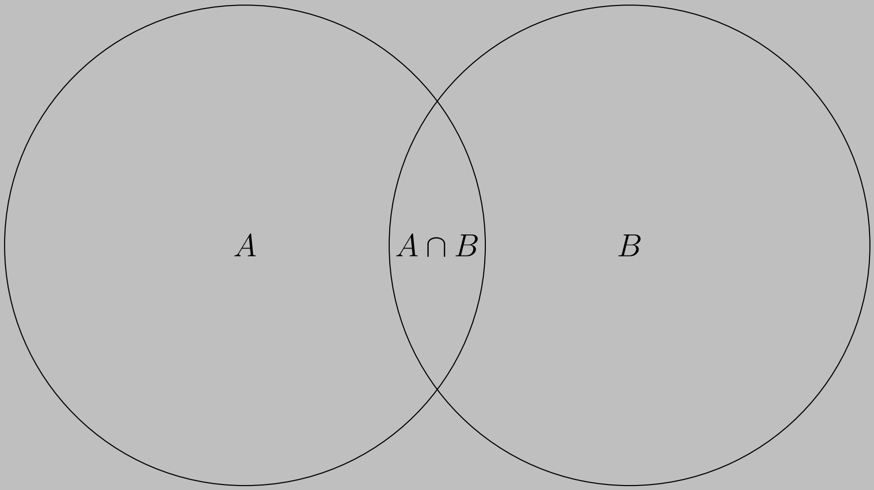
$$\begin{aligned} 2 &\in \{1,2,3\} \\ \{1,2\} &\subset \{1,2,3\} \\ \{1,2\}\cup\{2,3\} &= \{1,2,3\} \\ \{1,2\}\cap\{2,3\} &= \{2\} \end{aligned}$$

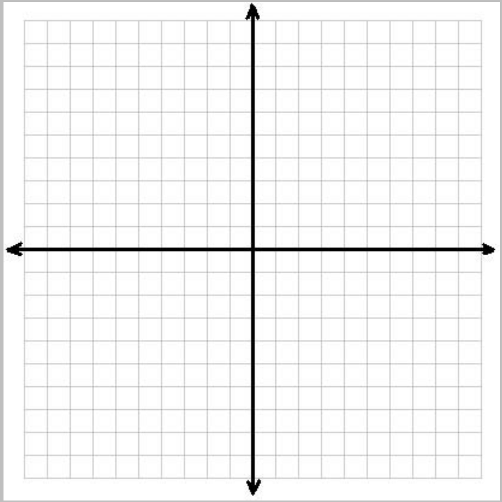
Venn Diagrams



The entire diagram represents $A \cup B$

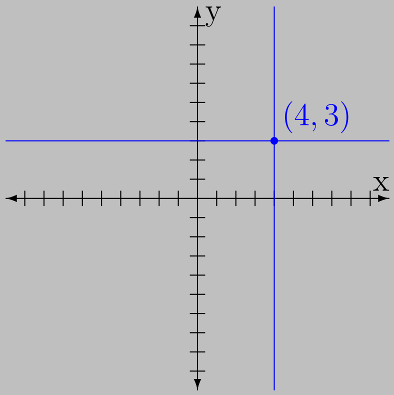
Graphing

The horizontal line is the x axis.
The vertical line is the y axis.
Where the axes cross is called the origin.
Every point on the grid corresponds to an (x, y) coordinate pair.
The coordinate pair for the origin is $(0, 0)$.
Positive x values are to the right of the origin.
Positive y values are above the origin.



Graphing a Point

To graph the point $(4, 3)$ one traces a vertical line at $x = 4$ and a horizontal line at $y = 3$. The point is located where the two lines cross.



Graphing a Line

Do **not** freehand a line on graph paper which looks "sort of" like the line shown on the calculator.

Use the **TABLE** feature to locate (x, y) coordinate pairs.
Choose 2 points and place them on the grid.
List the (x, y) coordinates used.
Draw a line through the 2 points using a straight edge.

x	y
-4	-8
4	4