Properties of a Finite Universe

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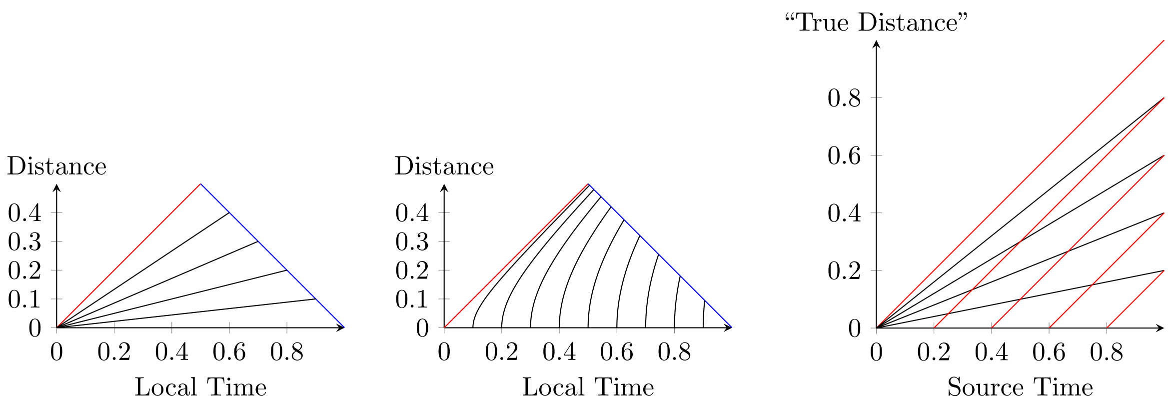
Hubble discovered that galaxies are moving away from us at speeds proportional to their distances, at least over the range of distances for which one can observe Cepheid variable stars. Assuming relatively constant velocities over time, this is what the universe would look like if it had exploded 13.7 billion years ago, more or less. Hubble's Law breaks down at large distances unless you use "true distances", which are the hypothetical distances to where objects would be now. Interestingly, it seems that the inverse square law also uses "true distances."

When you look at the sky you are looking at the past, so how far you can see depends on how large the universe was at the time. This is illustrated by the following diagrams, where time is given as a fraction of the total age of the universe (T) and distance as fractions of cT.

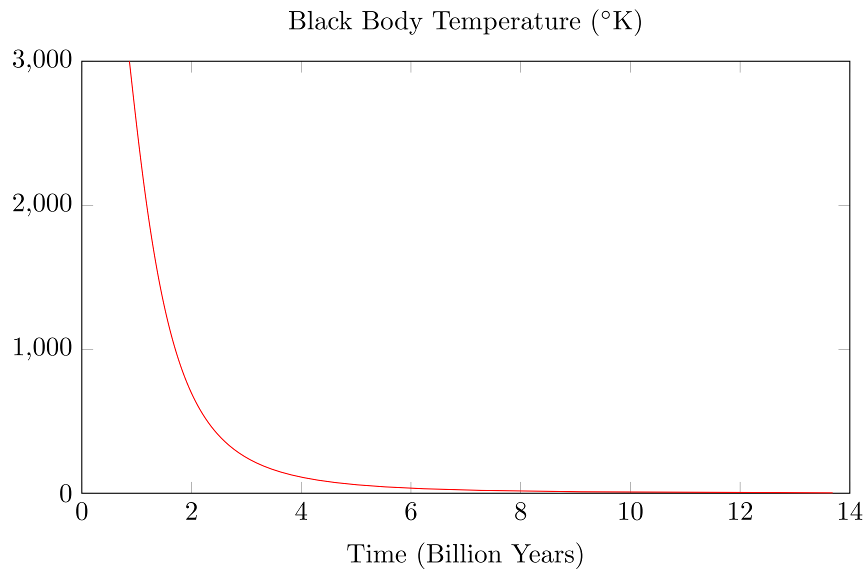


The blue line corresponds to the observed universe. Everything we see in the sky lies on that line. The other lines in the first graph represent the expanding universe, and the red line represents the upper limit on the size of the universe imposed by the speed of light. The curves in the second graph mark the age of the universe due to time dilatation. The faster an object moves, the slower its clock runs (relative to us). The last graph shows light (red lines) traveling from one location to the rest of the expanding universe.

The next major discovery was cosmic microwave background radiation. Once you remove the effects of our own galaxy and its velocity, this radiation is uniform in all directions, corresponding to a black body radiation of about 3° Kelvin. This radiation is supposed to come from when the universe first cooled to about 3000° Kelvin and went from being a hot opaque plasma to a hot transparent gas. The red shift needed to achieve this effect corresponds to a velocity of $v = 0.992c$. The corresponding date of the cosmic background event is $0.0634\,T$ or about 868 million years after the big bang (assuming $T = 13.7$ billion years).

After the cosmic background event, the universe would have remained an oven. A hypothetical observer would have seen an expanding transparent sphere surrounded by hot opaque plasma. Black body radiation depends only on the temperature, not the size of the enclosure, so it would only cool due to red shift. (See graph.)

If the universe had a center, we should be able to locate it. Assuming the universe has a positive mass, everything closer to the center would appear red shifted and everything further from the center would appear blue shifted. At the very least we should expect some sort of non-uniform distribution of galaxies with respect to distance from the center. Finally, if you run the clock backward, at some point the universe was dense enough to have formed a black hole. Consequently, we assume a universe with no center or edges, where every observer sees themselves as being at the center.



At this point physicists generally start torturing general relatively to warp the universe into a shape which matches our observations. Frankly, it can't be done. General relativity was developed to explain gravitation, which always has a center and extends to infinity. If anything, general relativity warps the universe the wrong way (concave verses convex).

So what sort of shape has no center and is finite in size? The usual analogy is that the universe is like the surface of a sphere. Taking this literally, let us assume a fourth dimensional space with dimensions d_1 through d_4, where the surface of a hypersphere is given by

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 = R^2 \tag{1}$$

where R is the radius of the hypersphere. The hypersphere can be expressed in (x, y, z) coordinates using

$$\begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} \\ d_1 &= R\sin(r/R)x/r \\ d_2 &= R\sin(r/R)y/r \\ d_3 &= R\sin(r/R)z/r \\ d_4 &= R\cos(r/R) \end{aligned}$$

and the surface distance from any point on the hypersphere to its antipode is πR. Note that $d_1 \to x$, $d_2 \to y$ and $d_3 \to z$ as $r/R \to 0$.

More importantly, a sphere of radius r will behave exaxtly like a sphere of radius $R\sin(r/R)$, with surface area

$$A = 4\pi R^2 \sin^2(r/R) \quad .$$

Since the distance from a point to its antipode is the same in every direction, every point on a sphere is equidistant to the center and equidistant to the antipode of the center, which by definition is also a sphere.

A small hypersphere ($\pi R \ll cT$) will possess a number of interesting optical characteristics. The inverse square law breaks down at large distances. At some point objects will even start appearing larger and brighter rather than smaller and dimmer. Every galaxy we see beyond πR would be an image of some closer galaxy, generally from the opposite side of the sky.

A more likely case is that the universe is expanding close to the speed of light (closer than $0.992c$). This would mean that everything we see in the sky is real, not an image of something else. However, there should still be observable effects on the inverse square law.

Fortunately, this model works well with cosmic black body radiation. When we look at the cosmic background event, we are only seeing a small sphere of hot plasma surrounding the antipode (much smaller than $0.0634cT$). However, all of these photons are being focused on us. Other points in space receive their photons from a completely different sphere of hot plasma.

It should also be noted that gravity or any force with an inverse square characteristic will neither help nor hinder the expansion of the universe. Since every observer sees themselves to be at the center of the universe, the global effects cancel out. No force means potential energy doesn't change.

Appendix

The age of the universe graph was computed parametrically over v (actually, v/c) using

$$\begin{aligned} t &= \frac{t_0}{\sqrt{1 - v^2/c^2}} \\ d &= \frac{vt_0}{\sqrt{1 - v^2/c^2}} \end{aligned}$$

for

$$v \leq c\sqrt{(1 - t_0^2)/(1 + t_0^2)}$$

and $t_0 \in \{0.1T, \ldots, 0.9T\}$.

The velocity for the cosmic background radiation source was found using

$$\frac{3^\circ}{3000^\circ} = (1 - v/c)\sqrt{1 - v^2/c^2}$$

and solving numerically (no closed form inverse). The time of the cosmic background event was found using

$$\begin{aligned} vt_o &= (T - t_o)c \\ t_o &= T/(1 + v/c) \end{aligned}$$

then converting to the source time using

$$\begin{aligned} t_s &= t_o\sqrt{1 - v^2/c^2} \\ &= T\sqrt{\frac{1 - v/c}{1 + v/c}} \end{aligned}$$

for $v = 0.992c$.

The black body temperature graph was also computed parametrically over v, using

$$\begin{aligned} t &= 0.868 \times 10^9\sqrt{\frac{1 + v/c}{1 - v/c}} \quad \text{(years)} \\ y &= 3000(1 - v/c)\sqrt{1 - v^2/c^2} \quad (^\circ\text{K}) \end{aligned}$$

using y for temperature.